Chapter 6

Problems: 4, 5, 12, 23, 24, 35, 38, 40, 49, 62 and 68
Think about: 11, 45 and 52

4 • By what factor does the kinetic energy of a particle change if its speed is doubled but its mass is cut in half?

Determine the Concept The kinetic energy of a particle is proportional to the square of its speed. Because $K = \frac{1}{2}mv^2$, replacing v by $2v$ and m by $\frac{1}{2}\boldsymbol{m}$ yields

$$\boldsymbol{K'} = \tfrac{1}{2}\left(\tfrac{1}{2}\boldsymbol{m}\right)(2\boldsymbol{v})^2 = 2\left(\tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}^2\right) = 2\boldsymbol{K}\,.$$

Thus doubling the speed of a particle and halving its mass doubles its kinetic energy.

5 • Give an example of a particle that has constant kinetic energy but is accelerating. Can a non-accelerating particle have a changing kinetic energy? If so, give an example.

Determine the Concept

✓ A particle moving along a circular path at constant speed has constant kinetic energy but is accelerating (because its velocity is continually changing).

✓ No, because if the particle is not accelerating, the net force acting on it must be zero and, consequently, its kinetic energy must be constant.

11 • True or false: (*a*) The gravitational force cannot do work on an object, because it is not a contact force. (*b*) Static friction can never do work on an object. (*c*) As a negatively charged electron in an atom is pulled from a positively charged nucleus, the electric force on the electron does work that has a positive value. (*d*) If a particle is moving along a circular path, the total work being done on it is necessarily zero.

(*a*) False. The definition of work is not limited to displacements caused by contact forces. Consider the work done by the gravitational force on an object in freefall.

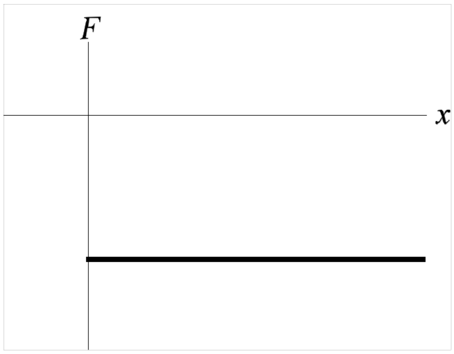
(*b*) False. The force that the earth exerts on your foot as you walk (or run) is a static friction force. Another example is the force that the earth exerts on the tire mounted on a drive wheel of an automobile.

(*c*) False. The direction of the electric force on the electron during its removal from the atom is opposite that of the displacement of the electron.

(*d*) False. If the particle is accelerating, there must be a net force acting on it and, hence, work is done on it.

12 •• A hockey puck has an initial velocity in the $+x$ direction on a horizontal sheet of ice. Qualitatively sketch the force-versus-position graph for the (constant) horizontal force that would need to act on the puck to bring it to rest. Assume that the puck is located at $x = 0$ when the force begins to act. Show that the sign of the area under the curve agrees with the sign of the change in the puck's kinetic energy and interpret this in terms of the work–kinetic–energy theorem.

Determine the Concept The graph of the force F acting on the puck as a function of its position x is shown to the right. Note that, because the force is negative, the area bounded by it and the x axis is negative and, hence, the net work done by the force is negative. In accordance with the work-kinetic energy theorem, the change in kinetic energy is negative and the puck loses all of its initial kinetic energy.



23 • Find the kinetic energy of (*a*) a 0.145-kg baseball moving with a speed of 45.0 m/s, and (*b*) a 60.0-kg jogger running at a steady pace of 9.00 min/mi.

Picture the Problem We can use $\frac{1}{2}mv^2$ to find the kinetic energy of the baseball and the jogger.

(*a*) Use the definition of K to find the kinetic energy of the baseball:

$$K_{\text{baseball}} = \tfrac{1}{2}(0.145\,\text{kg})(45.0\,\text{m/s})^2 = \boxed{147\,\text{J}}$$

(*b*) Convert the jogger's pace of 9.00 min/mi into a speed:

$$v = \left(\frac{1\,\text{mi}}{9.00\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{1609\,\text{m}}{1\,\text{mi}}\right) = 2.98\,\text{m/s}$$

Use the definition of K to find the jogger's kinetic energy:

$$K_{\text{jogger}} = \tfrac{1}{2}(60.0\,\text{kg})(2.98\,\text{m/s})^2 = \boxed{266\,\text{J}}$$

24 • A 6.0-kg box is raised a distance of 3.0 m from rest by a vertical applied force of 80 N. Find (*a*) the work done on the box by the applied force, (*b*) the work done on the box by gravity, and (*c*) the final kinetic energy of the box.

Picture the Problem The work done by the force acting on the box is the scalar product of the force producing the displacement and the displacement of the box. Because the weight of an object is the gravitational force acting on it and this force acts downward, the work done by gravity is negative. We can use the work-kinetic energy theorem to find the final kinetic energy of the box.

(*a*) Use the definition of work to obtain:

$$W_{\text{on the box}} = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{y}} \qquad (1)$$

$\vec{F}$ and $\Delta\vec{y}$ are given by:

$$\vec{F} = (80\ \text{N})\hat{j}$$

and

$$\Delta\vec{y} = (3.0\ \text{m})\hat{j}$$

Substitute for $\vec{F}$ and $\Delta\vec{y}$ in equation (1) and evaluate $W_{\text{on the box}}$:

$$W_{\text{on the box}} = (80\ \text{N})\hat{j}\cdot(3.0\ \text{m})\hat{j} = \boxed{0.24\ \text{kJ}}$$

(*b*) Apply the definition of work again to obtain:

$$W_{\text{by gravity}} = \vec{\mathbf{F}}_{\text{g}}\cdot\Delta\vec{\mathbf{y}} \qquad (2)$$

$\vec{F}$ and $\Delta\vec{y}$ are given by:

$$\vec{F}_{\text{g}} = -mg\hat{j} = -(6.0\ \text{kg})(9.81\ \text{m/s}^2)\hat{j} = (-58.9\ \text{N})\hat{j}$$

and

$$\Delta\vec{y} = (3.0\ \text{m})\hat{j}$$

Substituting for $\vec{F}$ and $\Delta\vec{y}$ and simplifying yields:

$$W_{\text{by gravity}} = (-58.9\ \text{N})\hat{j}\cdot(3.0\ \text{m})\hat{j} = \boxed{-0.18\ \text{kJ}}$$

(*c*) According to the work-kinetic energy theorem:

$$\Delta K = W_{\text{on the box}} + W_{\text{by gravity}}$$

or, because $K_{\text{i}} = 0$,

$$K_{\text{f}} = W_{\text{on the box}} + W_{\text{by gravity}}$$

Substitute numerical values and evaluate K_{f}:

$$K_{\text{f}} = 0.24\ \text{kJ} - 0.18\ \text{kJ} = \boxed{0.06\ \text{kJ}}$$

35 •• Particle a has mass m, is initially located on the positive x axis at $x = x_0$ and is subject to a repulsive force F_x from particle b. The location of particle b is fixed at the origin. The force F_x is inversely proportional to the square of the distance x between the particles. That is, $F_x = A/x^2$, where A is a positive constant. Particle a is released from rest and allowed to move under the influence of the force. Find an expression for the work done by the force on a as a function of x. Find both the kinetic energy and speed of a as x approaches infinity.

Picture the Problem Because the force varies inversely with distance, we need to use the integral form of the expression for the work done on the particle. We can then apply the work-kinetic energy theorem to the particle to find its kinetic energy and speed as functions of x.

The work done by the force on the particle is given by:

$$W_{x_0\to x} = \int_{x_0}^{x} \vec{F}\cdot dx\hat{i} = \int_{x_0}^{x} F(\cos\theta)dx$$

where θ is the angle between $\vec{F}$ and $dx\hat{i}$.

Substituting for F and noting that $\theta = 0$:

$$W_{x_0\to x} = \int_{x_0}^{x} \frac{A}{x^2}(\cos 0°)dx = A\int_{x_0}^{x} \frac{1}{x^2}dx$$

Evaluate this integral to obtain:

$$W_{x_0\to x} = -A\left[\frac{1}{x}\right]_{x_0}^{x} = \boxed{\frac{A}{x_0} - \frac{A}{x}}$$

Applying the work-kinetic energy theorem to the particle yields:

$$W_{x_0\to x} = \Delta K = K_x - K_{x_0}$$

or, because the particle is released from rest at $x = x_0$,

$$W_{x_0\to x} = K_x$$

Substitute for $W_{x_0\to x}$ to obtain:

$$K_x = \frac{A}{x_0} - \frac{A}{x}$$

As x → ∞:

$$K_{x\to\infty} = \boxed{\frac{A}{x_0}}$$

and

$$v_{x\to\infty} = \sqrt{\frac{2K_{x\to\infty}}{m}} = \sqrt{\frac{2\left(\frac{A}{x_0}\right)}{m}} = \boxed{\sqrt{\frac{2A}{mx_0}}}$$

38 • Two vectors $\vec{A}$ and $\vec{B}$ each have magnitudes of 6.0 m and the angle between their directions is 60º. Find $\vec{A}\cdot\vec{B}$.

Picture the Problem We can use the definition of the scalar product to evaluate $\vec{A}\cdot\vec{B}$.

Express the definition of the scalar product $\vec{A}\cdot\vec{B}$:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Substitute numerical values and evaluate $\vec{A}\cdot\vec{B}$:

$$\vec{A}\cdot\vec{B} = (6.0\,\text{m})(6.0\,\text{m})\cos 60^\circ = \boxed{18\,\text{m}^2}$$

40 • Find the angles between the vectors $\vec{A}$ and $\vec{B}$ given: (*a*) $\vec{A} = 3\hat{i} - 6\hat{j}$, $\vec{B} = -4\hat{i} + 2\hat{j}$; (*b*) $\vec{A} = 5\hat{i} + 5\hat{j}$, $\vec{B} = 2\hat{i} - 4\hat{j}$; and (*c*) $\vec{A} = 6\hat{i} + 4\hat{j}$, $\vec{B} = 4\hat{i} - 6\hat{j}$.

Picture the Problem The scalar product of two-dimensional vectors $\vec{A}$ and $\vec{B}$ is $AB\cos\theta = A_xB_x + A_yB_y$. Hence the angle between vectors $\vec{A}$ and $\vec{B}$ is given by

$$\theta = \cos^{-1}\left(\frac{A_xB_x + A_yB_y}{AB}\right).$$

(*a*) For $\vec{A} = 3\hat{i} - 6\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$:

$$\theta = \cos^{-1}\left(\frac{(3)(-4)+(-6)(2)}{\sqrt{3^2+(-6)^2}\sqrt{(-4)^2+(2)^2}}\right) = \boxed{143^\circ}$$

(*b*) For $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{j}$:

$$\theta = \cos^{-1}\left(\frac{(5)(2)+(5)(-4)}{\sqrt{5^2+5^2}\sqrt{2^2+(-4)^2}}\right) = \boxed{108^\circ}$$

(*c*) For $\vec{A} = 6\hat{i} + 4\hat{j}$ and $\vec{B} = 4\hat{i} - 6\hat{j}$:

$$\theta = \cos^{-1}\left(\frac{(6)(4)+(4)(-6)}{\sqrt{6^2+4^2}\sqrt{4^2+(-6)^2}}\right) = \boxed{90^\circ}$$

45 •• In Chapter 8, we shall introduce a new vector for a particle, called its *linear momentum*, symbolized by $\vec{\boldsymbol{p}}$. Mathematically, it is related to the mass m and velocity $\vec{\mathbf{v}}$ of the particle by $\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$. (*a*) Show that the particle's kinetic energy K can be expressed as $\boldsymbol{K} = \frac{\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{p}}}{2\boldsymbol{m}}$. (*b*) Compute the linear momentum of a particle of mass 2.5 kg that is moving at a speed of 15 m/s at an angle of 25° clockwise from the +*x* axis in the *xy* plane. (*c*) Compute its kinetic energy using both $K = \frac{1}{2}mv^2$ and $\boldsymbol{K} = \frac{\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{p}}}{2\boldsymbol{m}}$ and verify that they give the same result.

Picture the Problem The scalar product of two vectors is the product of their magnitudes multiplied by the cosine of the angle between them.

(*a*) We're to show that:

$$\boldsymbol{K} = \frac{\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{p}}}{2\boldsymbol{m}} \qquad (1)$$

The scalar product of $\vec{\boldsymbol{p}}$ with itself is:

$$\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{p}} = \boldsymbol{p}^2$$

Because $p = mv$:

$$\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{p}} = \boldsymbol{m}^2\boldsymbol{v}^2$$

Substitute in equation (1) and simplify to obtain:

$$K = \frac{m^2v^2}{2m} = \boxed{\tfrac{1}{2}mv^2}$$

(*b*) The linear momentum of the particle is given by:

$$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$$

Substitute for m and $\vec{\boldsymbol{v}}$ and simplify to obtain:

$$\begin{aligned}\vec{\boldsymbol{p}} &= (2.5\ \text{kg})\left[(15\ \text{m/s})\cos 335°\,\hat{\boldsymbol{i}} + (15\ \text{m/s})\sin 335°\,\hat{\boldsymbol{j}}\right]\\ &= \boxed{(34\ \text{kg}\cdot\text{m/s})\hat{\boldsymbol{i}} + (-16\ \text{kg}\cdot\text{m/s})\hat{\boldsymbol{j}}}\end{aligned}$$

(*c*) Evaluating $K = \frac{1}{2}mv^2$ yields:

$$K = \tfrac{1}{2}(2.5\ \text{kg})(15\ \text{m/s})^2 = \boxed{0.28\ \text{kJ}}$$

Evaluating $\boldsymbol{K} = \frac{\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{p}}}{2\boldsymbol{m}}$ yields:

$$\begin{aligned}\boldsymbol{K} &= \frac{\left[(34\ \text{m/s})\hat{\boldsymbol{i}} + (-16\ \text{m/s})\hat{\boldsymbol{j}}\right]\cdot\left[(34\ \text{m/s})\hat{\boldsymbol{i}} + (-16\ \text{m/s})\hat{\boldsymbol{j}}\right]}{2(2.5\ \text{kg})} = \frac{(34\ \text{m/s})^2 + (-16\ \text{m/s})^2}{5.0\ \text{kg}}\\ &= \boxed{0.28\ \text{kJ}}\end{aligned}$$

49 • A single force of 5.0 N in the + x direction acts on an 8.0-kg object. (*a*) If the object starts from rest at $x = 0$ at time $t = 0$, write an expression for the power delivered by this force as a function of time. (*b)* What is the power delivered by this force at time $t = 3.0$ s?

Picture the Problem We can use Newton's 2nd law and the definition of acceleration to express the velocity of this object as a function of time. The power input of the force accelerating the object is defined to be the rate at which it does work; that is, $P = dW/dt = \vec{F}\cdot\vec{v}$.

(*a*) The power of the force as a function of time is given by:

$$P(t) = \frac{dW}{dt} = \vec{F}\cdot\vec{v} = Fv(t)\cos\theta$$

or, because the force acts in the same direction as the velocity of the object,

$$P(t) = Fv(t) \qquad (1)$$

Express the velocity of the object as a function of its acceleration and time:

$$v(t) = at$$

Applying $\sum\vec{F} = m\vec{a}$ to the object yields:

$$a = \frac{F}{m}$$

Substitute for a in the expression for v to obtain:

$$v(t) = \frac{F}{m}t$$

Substituting in equation (1) yields:

$$P(t) = \frac{F^2}{m}t$$

Substitute numerical values and evaluate $P(t)$:

$$P(t) = \frac{(5.0\ \text{N})^2}{8.0\ \text{kg}}t = (3.125\ \text{W/s})t$$
$$= \boxed{(3.1\ \text{W/s})t}$$

(*b*) Evaluate P(3.0 s):

$$P(3.0\ \text{s}) = (3.125\ \text{W/s})(3.0\ \text{s}) = \boxed{9.4\ \text{W}}$$

52 •• A cannon placed at the edge of a cliff of height H fires a cannonball directly upward with an initial speed v_0. The cannonball rises, falls back down (missing the cannon by a small margin) and lands at the foot of the cliff. Neglecting air resistance, calculate the velocity $\vec{v}$ as a function of time, and show explicitly that the integral of $\vec{F}_{\text{net}}\cdot\vec{v}$ over the time that the cannonball spends in flight is equal to the change in the kinetic energy of the cannonball over the same time.

Picture the Problem Because, in the absence of air resistance, the acceleration of the cannonball is constant, we can use a constant-acceleration equation to relate its velocity to the time it has been in flight. We can apply Newton's 2nd law to the cannonball to find the net force acting on it and then form the dot product of $\vec{F}$ and $\vec{v}$ to express the rate at which the gravitational field does work on the cannonball. Integrating this expression over the time-of-flight T of the ball will yield the desired result.

Express the velocity of the cannonball as a function of time while it is in the air:

$$\vec{v}(t) = 0\hat{i} + (v_0 - gt)\hat{j} = \boxed{(v_0 - gt)\hat{j}}$$

Apply $\sum\vec{F} = m\vec{a}$ to the cannonball to express the force acting on it while it is in the air:

$$\vec{F}_{\text{net}} = -mg\,\hat{j}$$

Evaluate $\vec{F}_{\text{net}}\cdot\vec{v}$:

$$\begin{aligned}\vec{F}_{\text{net}}\cdot\vec{v} &= -mg\,\hat{j}\cdot(v_0 - gt)\hat{j}\\ &= -mgv_0 + mg^2t\end{aligned}$$

Relate $\vec{F}_{\text{net}}\cdot\vec{v}$ to the rate at which work is being done on the cannonball:

$$\frac{dW}{dt} = \vec{F}_{\text{net}}\cdot\vec{v} = -mgv_0 + mg^2t$$

Separate the variables and integrate over the time T that the cannonball is in the air:

$$\begin{aligned}W &= \int_0^T\left(-mgv_0 + mg^2t\right)dt\\ &= \tfrac{1}{2}mg^2T^2 - mgv_0T\end{aligned} \qquad (1)$$

Using a constant-acceleration equation, relate the time-of-flight T to the initial and impact speeds of the cannonball:

$$v = v_0 - gT \Rightarrow T = \frac{v_0 - v}{g}$$

Substitute for T in equation (1) and simplify to evaluate W:

$$W = \tfrac{1}{2}mg^2\frac{v_0^2 - 2vv_0 + v^2}{g^2} - mgv_0\left(\frac{v_0 - v}{g}\right) = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = \boxed{\Delta K}$$

62 •• The magnitude of the single force acting on a particle of mass m is given by $F = bx^2$, where b is a constant. The particle starts from rest. After it travels a distance L, determine its (*a*) kinetic energy and (*b*) speed.

Picture the Problem We can apply the work-kinetic energy theorem and the definition of work to the particle to find its kinetic energy when it has traveled a distance L. We can then use the definition of kinetic energy to find its speed at this location.

(*a*) Apply the work-kinetic energy theorem to the particle to obtain:

$$W_{\text{total}} = \Delta K = K_{\text{f}} - K_{\text{i}}$$

or, because the particle is initially at rest,

$$W_{\text{total}} = K_{\text{f}} = \tfrac{1}{2}mv_{\text{f}}^2 \qquad (1)$$

The total work done on the particle is also given by:

$$\boldsymbol{dW_{\text{total}} = \vec{F}\cdot d\vec{x} = Fdx}\cos\boldsymbol{\theta}$$

or, because $\vec{F}$ and $d\vec{x}$ are in the same direction,

$$dW_{\text{total}} = K_{\text{f}} = Fdx$$

Substituting for F_x and going from differential to integral form yields:

$$\boldsymbol{K_{\text{f}} = \int_0^L bx^2dx = b\int_0^L x^2dx = \boxed{\tfrac{1}{3}bL^3}}$$

(*b*) Substitute for K_{f} in equation (1) to obtain:

$$\tfrac{1}{3}bL^3 = \tfrac{1}{2}mv_{\text{f}}^2 \Rightarrow v_{\text{f}} = \boxed{\sqrt{\frac{2bL^3}{3m}}}$$

68 •• The force F_x acting on a 0.500-kg particle is shown as a function of x in Figure 6-36. (*a*) From the graph, calculate the work done by the force when the particle moves from $x = 0.00$ to the following values of x: –4.00, –3.00, –2.00, –1.00, +1.00, +2.00, +3.00, and +4.00 m. (*b*) If it starts with a velocity of 2.00 m/s in the $+x$ direction, how far will the particle go in that direction before stopping?

Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, F is positive, so W is negative for $x < 0$.

(*a*) Use either the formulas for the areas of simple geometric figures *or* counting squares and multiplying by the work represented by one square (each square is 1.00 J) to complete the following table:

x, m	−4.00	−3.00	−2.00	−1.00	1.00	2.00	3.00	4.00
W, J	−11.4	−10.2	−7.0	−3.0	1.0	0	−2.0	−3.0

(*b*) The energy that the particle must lose before it stops is its kinetic energy at $x = 0$:

$$K_{x=0} = \tfrac{1}{2}mv_{x=0}^2$$

Substitute numerical values and evaluate $K_{x=0}$:

$$\boldsymbol{K}_{x=0} = \tfrac{1}{2}(0.500\text{ kg})(2.00\text{ m/s})^2 = 1.00\text{ J}$$

The particle is accelerated (recall that $F_x = -dU/dx$) between $x = 0$ and $x = 1.00$ m and gains an additional 1.00 J of kinetic energy. When it arrives at $x = 2.00$ m, however, it will have lost the kinetic energy it gained between $x = 0$ and $x = 1.00$ m and its kinetic energy will again be 1.00 J. Inspection of the graph leads us to conclude that it will have lost its remaining kinetic energy when it reaches $\boldsymbol{x} = \boxed{2.50\text{ m.}}$